Transverse surface waves in a functionally graded piezoelectric substrate coated with a finite-thickness metal waveguide layer

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An analytical study on transverse surface waves propagating in a functionally graded piezoelectric substrate carrying a metal layer of finite thickness was carried out. Dispersion relations for the existence of the waves were obtained and the effects of material gradient on wave propagation were quantified. Numerical examples show that the presence of the material gradient affects significantly the fundamental mode but has only negligible effects on the higher order modes. Depending on whether the surface wave velocity is smaller or greater than the bulk shear wave velocity in the metal layer, three different types of the dispersion behavior are discussed. © 2009 American Institute of Physics.

Transverse surface waves in piezoelectric materials and structures are attractive for designing signal-processing devices due to their high performance and simple particle motion.1 One type of the transverse surface wave is the Love wave, which is always dispersive and has more than one mode. The Love wave has been extensively studied and used in surface acoustic wave (SAW) sensors, filters, delay lines, and the like.2–9

Another type of the transverse surface wave is the Bleustein–Gulyaev (BG) wave, which exists and propagates only at the free surface of a piezoelectric half-space.10,11 The BG wave is closely related to the bulk shear wave and, although seldom used in practice at present, may have certain advantages over the currently used SAW devices due to its simpler particle motion.

Recently, to improve the efficiency and natural life of the SAW devices, the potential application of functionally graded piezoelectric materials (FGPMs) as SAW substrates has been explored.12–15 Curtis and Redwood16 carried out a theoretical study on the propagation of transverse surface waves in a nongradient piezoelectric substrate carrying a finite-thickness metal layer. Such a wave is related both to the BG wave and the Love wave, and hence would have great importance in practical applications. However, to the best of our knowledge, there exists no study on transverse surface waves in a FGPM substrate coated with a metal layer of finite thickness.

We consider in the present letter a FGPM, here taken to be of class 6 mm (or $\infty$), occupying the half-space $x>0$, with its polar axis oriented along the z direction of the Cartesian coordinates $(x,y,z)$. It is assumed that the physical properties of the FGPM change gradually along the x direction only. Let a metal layer of uniform thickness h be deposited perfectly on the FGPM substrate, resulting in a surface at $z=-h$ free from external forces. Without loss of generality, it is assumed that the waves propagate in the positive direc-

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tion of the y-axis, such that the nonzero field quantities representing the motion are only functions of the coordinates $(x,y)$ and time $t$.

Let $w$ and $\varphi$ denote separately the mechanical displacement and the electrical potential function of the FGPM substrate. The coupled field equations are given by

$$c_{44,z} w_x + c_{44} \nabla^2 w + e_{15,z} \varphi_x + e_{15} \nabla^2 \varphi = \rho \ddot{w},$$

$$e_{15,z} w_x + e_{15} \nabla^2 w - e_{11,zz} \varphi_x - e_{11} \nabla^2 \varphi = 0,$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $c_{44}, e_{15}, e_{11},$ and $\rho$ are the elastic, piezoelectric, dielectric constants, and mass density of the substrate, respectively, which are all functions of $x$ (i.e., the substrate is functionally graded). For simplicity, all the material parameters of the substrate are taken to have the same exponential function variation as

$$c_{44}(x) = c_{44}^0 e^{\alpha x}, \quad e_{15}(x) = e_{15}^0 e^{\alpha x},$$

$$e_{11}(x) = e_{11}^0 e^{\alpha x}, \quad \rho(x) = \rho_0 e^{\alpha x},$$

where $\alpha$ is the coefficient characterizing the profile of the material gradient and the superscript “0” is used to denote the values of material parameters at $x=0$. Substitution of Eq. (2) into Eq. (1) yields

$$c_{44}^0 (\alpha w_x + \nabla^2 w) + e_{15}^0 (\alpha \varphi_x + \nabla^2 \varphi) = \rho \ddot{w},$$

$$e_{11}^0 (\alpha \varphi_x + \nabla^2 \varphi) - e_{11}^0 (\alpha w_x + \nabla^2 w) = 0.$$  

For the metal layer of finite thickness, let $w'$ denote its mechanical displacement in the z direction. The governing field equation is

$$\nabla^2 w' - (1/c_1^2) w'' = 0,$$

where $c_l = (c_{44}^l/\rho_l)^{1/2}$ is the bulk shear wave velocity, with $c_{44}^l$ and $\rho_l$ representing the shear modulus and mass density of the metal layer, respectively.

The boundary and continuity conditions are (1) $\sigma_x' = 0$ at $x=-h$; (2) $w = w''$, $\sigma_x = \sigma_x'$, $\varphi = 0$ at $x=0$; and (3) $w$ and $\varphi$ → 0 as $x \rightarrow \infty$. 

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Built upon our previous work, we consider the following transverse waves satisfying attenuation condition (3):

\[
\begin{align*}
  w(x, y, t) &= A_1 e^{\alpha x} \exp[i k(y - ct)] \\
  \varphi(x, y, t) &= [A_2 e^{\alpha x} + (e^{i \psi} / e_{11}^{0}) A_1 e^{\alpha x}] \exp[i k(y - ct)] \\
  w' &= [A_3 e^{-b' x} + A_4 e^{b' x}] \exp[i k(y - ct)],
\end{align*}
\]

where \( b = (1 - c^2 / c_s^2)^{1/2} \) and \( c_s = [(c_{44}^0 + c_{15}^0 e_{15}^{0} / e_{11}^{0}) / \rho^{0}]^{1/2} \) is the bulk shear wave velocity in the substrate.

The assumption that \( \alpha < 0 \) is needed for the derivation of Eq. (5), which directly leads to the existence condition of transverse surface waves in the FGPM substrate, i.e., \( c < c_r \). The decay parameter \( b' \) in Eq. (6) is defined as

\[
b' = \sqrt{1 - c^2 / c_s^2} \quad \text{or} \quad b' = i \sqrt{c^2 / c_s^2 - 1},
\]

where \( c_s \) is the bulk shear wave velocity in the metal layer.

Substitution of Eqs. (5) and (6) and the corresponding stress components into the remaining boundary and continuity conditions (1) and (2) yields four linear, homogeneous algebraic equations for coefficients \( A_1, A_2, A_3, \) and \( A_4 \). For nontrivial solutions of these coefficients, the determinant of the coefficient matrix of the linear equations has to vanish. This leads to the following dispersion relation of the transverse waves:

\[
\bar{c}_{44} M + (e_{15}^{0})^2 / e_{11}^{0} N = c_4' 2 \pi H \sqrt{c_s^2 / c_s^2 - 1} \tan(2 \pi H \sqrt{c_s^2 / c_s^2 - 1}) = 0,
\]

where \( \bar{c}_{44} = c_{44}^0 + e_{15}^0 / e_{11}^0 \) is the equivalent shear modulus of the FGPM substrate.

In Eq. (9), \( b' \) cannot only take real values but also imaginary values, depending on whether the surface wave velocity \( c \) is smaller or greater than the bulk shear wave velocity in the metal layer, \( c_s \). Therefore, it is convenient to classify the present problem into three physical situations for which dispersion curves of different types may be found.\(^{16}\)

(a) Type 1: \( c_s > c_r > c_{BG} \), \( b' \) always real.
(b) Type 2: \( c_s > c_{BG} > c_r \), \( b' \) always imaginary.
(c) Type 3: \( c_s > c_r > c_{BG} \), \( b' \) real or imaginary.

Here, \( c_{BG} \) is the velocity of the BG wave on the surface of a piezoelectric substrate coated with an infinitely thin layer of conducting material.

For convenience, we introduce two dimensionless quantities: \( m = ah \) and \( H = h / \lambda \). Using Eqs. (7) and (8), we can then rewrite Eq. (9) separately for real \( b' \) and imaginary \( b' \) as

\[
\begin{align*}
\bar{c}_{44} M + (e_{15}^0)^2 / e_{11}^0 N &
- c_4' 2 \pi H \sqrt{1 - c_s^2 / c_s^2} \tan(2 \pi H \sqrt{1 - c_s^2 / c_s^2}) = 0, \\
\end{align*}
\]

\[
\text{FIG. 1. Phase velocity } c \text{ of type 1 transverse surface waves in the graded PZT-4 substrate carrying an aluminum layer for selected values of gradient coefficient: } H = h / \lambda, \ c_s = 2597 \text{ m/s, } c_{BG} = 2258 \text{ m/s, and } c_b = 2597 \text{ m/s}. \\
\]

\[
\text{FIG. 2. Phase velocity } c \text{ of type 2 transverse surface waves in graded PZT-4 substrate carrying a gold layer for selected values of gradient coefficient: } H = h / \lambda, \ c_s = 1222 \text{ m/s, } c_{2} = 2597 \text{ m/s, and } c_{BG} = 2258 \text{ m/s}. \\
\]
m=0 (i.e., a pure piezoelectric substrate) is altered by the presence of material gradient in the substrate. Not only the starting value of the mode is changed but also its cutoff frequency as the substrate becomes graded. For small values of α, the whole anomalous dispersion when α=0 is modified into partly normal dispersion and partly anomalous dispersion. On the other hand, if the gradient coefficient is sufficiently large, the mode is changed into totally normal dispersion.

Figure 2 shows that type 2 dispersive curves are closely related to the Love waves, which could exist even if the substrate material is nonpiezoelectric. The material gradient in the substrate has obvious effects on the phase velocity and cutoff frequency of the fundamental mode but negligible effects on the higher order modes. The starting value of the fundamental mode increases from a velocity of 2258 to 2597 m/s, i.e., at the bulk shear wave velocity of the zinc layer. These results suggest that the grading of the piezoelectric substrate creates a “new mode” to the fundamental mode, greatly expanding its velocity range, which is very interesting and appears new.

In summary, the effects of the material gradient in a piezoelectric substrate carrying a conducting layer on the dispersion behavior of transverse surface waves have been quantified analytically. It is demonstrated that the gradient significantly affects the fundamental mode of wave propagation but has only negligible effects on the higher order modes.

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