The energetics of dislocations accelerating and decelerating through the shear-wave speed barrier

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The question of whether a dislocation can accelerate through the shear-wave speed “barrier” is addressed by analyzing the transient motion at the instant when the velocity equals the shear-wave speed in the presence of acceleration. The stresses carried by the forming Mach wave fronts depend on the acceleration at this instant, and the energy required to push the dislocation through the shear-wave speed barrier is determined by means of the “contour-independent” dynamic $J$ integral, which defines the self-force on a moving defect, and is obtained as a function of the acceleration as it crosses the barrier. For decelerating motion through the shear-wave speed barrier this energy is released as dissipation. © 2009 American Institute of Physics. [DOI: 10.1063/1.3072351]

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The classical works of Weertman10–12 present the analysis of the stress field and the energetics of the steady state supersonic motion. While crossing of the barrier has been assumed prohibitive due to the $[c_2^2 - \dot{l}(t)]^{-1/2}$ singularity, which appears in the steady-state analysis, the present analysis aims at addressing the energetics of crossing the barrier by means of the transient analysis of a screw/edge dislocation in a general motion $x = \dot{l}(t)$ evaluated at the instant when the velocity $\dot{l}(t)$ equals the shear-wave speed in the presence of acceleration $\ddot{l}(t)$ and the evaluation of the self-force at this instant both for a Volterra and ramp-core model. In the transient analysis to the second-order terms that account for the acceleration, the term that yields $(c_2^2 - \dot{l})^2$ is not zero any longer but depends on the acceleration $\ddot{l}(t)$ at this instant.

The analysis performed evaluates the stress on the forming Mach cone which is the envelope of the wavelets emitted by the dislocation (Fig. 1) and determines a new singularity for a Volterra dislocation at this instant. General accelerating motion of dislocations has been analyzed by Eshelby13–17 also see Ref. 18 (for complete early references, see Ref. 12); however, crossing the shear-wave speed barrier has not been considered in the past and was recently analyzed by the authors.19 Here we consider the energetics at the instant as the dislocation accelerates and decelerates through the shear-wave speed, and we determine the self-force at this instant.

![FIG. 1. (Color online) Forming Mach cone for dislocation motion $l(t)$ = $x^2/t^2$ at $t=0.55$. The motion becomes supersonic at $t=0.5$. The ratio of the dislocation velocity at time $t=0.55$ vs the shear-wave speed is $l(t=0.55)/c_2=1.21$. The quantities are dimensionless and the speed $c_2$ is normalized to 1 for all figures.](image)
instant for a Volterra and ramp-core dislocation.

For a Volterra screw dislocation of Burgers vector $b$ with axis along the $y$ direction starting from rest at $t=0$ at the origin of the coordinate system $Oxyz$ to move along the $x$ direction according to $x=l(t)$, in a wave-front asymptotic analysis of the transient (rather than steady-state) radiated fields, the stress is a delta function on the Mach cone $(x,z,t^*)$, \[ \sigma_{yz}(x,z) = \mu \frac{\partial u}{\partial z} = \mu \frac{b}{2} \times \frac{z^2}{[(x-l(\tau))^2 + z^2 c_2^2]} \times [(1-c_2^2 l(\tau)^4)]^{1/2} \times \delta(t-t^*), \quad (1) \]

where $c_2$ is the shear-wave speed, $t^*(x,z)$ is the time at which the Mach front passes through the field point $(x,z)$, and $\tau(x,z)$ is the time of emission of a wavelet that has the time to reach and contribute to the point $(x,z,t^*)$ on the Mach cone; both $t^*$ and $\tau$ are determined from the equations defining the wave-front envelope and depend on the motion $l(t)$. The Mach cone $(x,z,t^*)$ is the envelope of wavelets (see Fig. 1) emitted by the dislocation on its path of supersonic motion (the subsonic ones do not form envelopes), and it starts forming at the instant when $l(t)=c_2$ at which point, in the analysis, the two complex conjugate roots $(\tau_0, \tau_1)$ of the function $f(\tau)=t-\tau-1/c_2 \sqrt{(x-l(\tau))^2 + z^2}$ (that determine the interval of the motion of the dislocation from which the emitted wavelets contribute to the front), coalesce into a double real one $\tau^*$, which subsequently splits into two real roots $\tau_0$ and $\tau_1$ and thus the Mach envelope forms. The contribution here is to analyze the fields and energies at this double root.

While Eq. (1) gives the stress on the Mach cone for $l(t)>c_2$, we perform a wave-front asymptotic analysis at the instant $t^*$ when the dislocation velocity $l(t^*)=c_2$ by expanding the motion $l(t)$ around this point according to \[ l(t) = l(t^*) + (t-t^*) c_2 + \frac{1}{2} (t-t^*)^2 l(t^*) + o(t-t^*)^2, \]

where $l(t^*)$ denotes the acceleration of the dislocation at the instant of crossing the shear-wave speed. For a Volterra dislocation, this wave-front asymptotic analysis gives, at the time when $l(t^*)=c_2$, the stress at points $(x,z)$ near the forming Mach cone obtained as the limit at the double root $\tau_0, \tau_1 \rightarrow \tau^*$, \[ \sigma_{yz} = \mu \lim_{\tau_0, \tau_1 \rightarrow \tau^*} \frac{b}{2} \tan^2 \theta \frac{c_2^{1/2} \ln[(\tau^* - \tau)]^{|\tau_0|}}{[l(\tau)]^{1/2}} \times [(t-t^*)^{1/2}]^{1/2} \times \delta(t-t^*), \quad (2) \]

with $\tan \theta = z/(x-l(\tau^*))$, $\sqrt{(x-l(\tau))^2 + z^2} = O(t-t^*)$, and $l(t^*)=c_2$.

The singularity in the coefficient of the delta function for the stress near the forming Mach cone in Eq. (2) is eliminated by applying a more physically realistic core model. The dislocation is modeled by a delta sequence arctan $[(x-l(t))/\varepsilon]$ ($\varepsilon \neq 0$) function rather than a delta function, so that the singularity is smoothed out by convolution. The same result to the leading order can be obtained by the theory of distributions. For this smoothing of the core, it is estimated that the stress on the forming Mach cone is about $1/30 \mu$, hence less than the theoretical strength of the material. For subsonic constant velocity dislocation motion, the stress obtained by the delta sequence coincides exactly [Ref. 21, Eq. (77)] with the constant velocity solution of a Peierls–Nabarro dislocation core model.

The required thermodynamic driving force, or self-force, needed to effectuate such motion is obtained by dynamic $J$ integral, based on Noether’s theorem expressing the requirement of invariance of the Lagrangian $W-T$ functional under an infinitesimal translation of the defect. The self-force is given by the contour independent expression

\[ F_I = \int_{\mathcal{S}} \frac{\partial}{\partial t} [\mu \dot{u}_{ij}] dV + \int_{\mathcal{S}} [(W-T) \delta_{ij} - u_{ij} \sigma_{ij}] dS, \quad (3) \]

where $u_i$ is the displacement, $V$ is a volume integral, and $S$ is a surface integral surrounding the moving dislocation and shrinking into it. Expression (3) applied to moving cracks coincides with the energy-release rate $G = F \times l(t)$, while for a general defect expression (3) can be interpreted as the difference in the work of the tractions to perform the celebrated Eshelby thought experiment in order to produce a translation of the defect, when comparing two motions differing by $\delta t$ for all times. For subsonic dislocation motion, Eq. (3) has been used to calculate the self-force and effective mass of a screw and edge dislocation. Here we evaluate the self-force at the instant of accelerating through the shear-wave speed barrier.

Expression (3) will yield for a ramp-core dislocation model the self-force by convolution,

\[ F_I(t) = \lim_{\varepsilon \rightarrow 0, x \rightarrow l(t)} \frac{1}{2 \pi} \frac{b}{|l(t)|^{1/2}} \times \int_{x=\varepsilon}^{\infty} \frac{z^2}{[(x-l(t))^2 + z^2]^{1/2}} \ln(x-l(t) - \xi) \times \frac{e}{e^2 + \xi} d\xi \times \delta(t-t^*), \quad (4) \]

which is integrable, and yields for the self-force a finite value (since $\varepsilon \neq 0$) in the coefficient of the delta function at the instant of crossing the barrier depending only on the acceleration of the dislocation at that instant. Alternatively, the integrals in Eq. (3) have been regularized according to the theory of distributions by a procedure that followed the one for subsonic motion.

The self-force given by Eq. (4), related to the energy-release rate $G$ required to be externally supplied for the dislocation to accelerate with $l(t^*)$ at the instant when its velocity $l(t^*)=c_2$, and it crosses the barrier, has a finite (for ramp-score parameter $\varepsilon \neq 0$) coefficient of the delta function, thus making the transition into the supersonic regime possible. It may be noted in Eq. (4) that for vanishing acceleration the self-force is infinite, consistent with the steady-state “relativistic” behavior.

The analysis is also performed in the case of a motion decelerating through the shear-wave speed into the subsonic regime. Now, the two distinct real roots coincide into a
double real, which splits into two complex conjugates and the dislocation detaches from the Mach front (Fig. 2). The sign in expression (2) is negative, which implies a positive self-force associated with the release of an energy flux, which is now dissipated into the material rather than required to be externally supplied.

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