Frequency and amplitude modulation in terahertz-sideband generation in quantum wells

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A narrow-band optical beam incident on a quantum well modulated at terahertz frequencies results in the formation of terahertz sidebands, i.e., optical signals at the input optical frequency plus integer multiples of the modulation frequency. The terahertz sidebands originate in the modulated complex optical susceptibility. In this study, we analyze theoretically terahertz-sideband generation in the context of frequency and amplitude modulations of the incident optical beam. We find that, at the optimum bias field, the frequency and amplitude modulations contribute almost equally to the maximum first-order sideband. The theoretical approach could offer simple yet useful tools for further engineering and optimization of modulator-type sideband-generation or wavelength-conversion devices. © 2009 American Institute of Physics. [DOI: 10.1063/1.3068494]

While semiconductor heterojunction quantum wells (QWs) have been studied widely for applications as optical-intensity modulators in radio-over-fiber and conventional digital optical-communication systems,\textsuperscript{1} which typically operate below 100 GHz, it was also demonstrated recently that they can possibly be used for ultrafast modulation in excess of 100 GHz and for terahertz sideband generation.\textsuperscript{2,3} There may be coherent effects\textsuperscript{4} that should not be ignored when the modulating frequency becomes comparable to the equivalent relaxation time of the optically excited states where the adiabatic approximation\textsuperscript{5} may break down. The basic operating principle for both cases is, however, the same: the change in optical properties in the QWs is caused by the applied modulating electric field, namely, the quantum-confined Stark effect (QCSE).\textsuperscript{6} Since the change in the interband optical properties of QWs due to the QCSE induces a corresponding change in the complex linear optical susceptibility, the field modulates not only the absorption coefficient but also the refractive index, which results in amplitude modulation (AM) and frequency modulation (FM), respectively, of an optical field transmitted through the QW. When a QW is used as an optical-intensity modulator, the modulation of the refractive index (i.e., FM) causes a typically unwanted frequency chirp. However, based on the study results, both AM and FM are found to contribute significantly in generating sidebands that are separated by the integer multiples of the modulating frequency. Although the physics of the sideband generation in QWs caused by the periodic modulation has been investigated experimentally\textsuperscript{7} and theoretically,\textsuperscript{8,9} a detailed analysis in the context of AM and FM has been lacking, which offers meaningful information for further engineering as well as elucidation of observed effects.

This study presents a systematic theoretical analysis of the effects of FM and AM of a narrow-band optical pulse on sideband generation in a periodically modulated multiple QW electroabsorption modulator (EAM), which also discusses its dependence on the modulation of the physical parameters of the material (optical susceptibility). The approach is based on conventional AM and narrow-band FM (NBFM) concepts that were developed in the field of communication systems.\textsuperscript{10} We employ an InP-based QW-EAM whose static (or low-frequency) optical properties at 300 K were investigated both experimentally and theoretically, which ensures that the current analysis is realistic.\textsuperscript{11}

The effect of FM on terahertz-sideband generations can be appreciated by analyzing the transmission of a near-infrared (NIR) temporal Gaussian optical pulse incident normal to the periodically modulated QW layers; the transmitted $E_T$ can be expressed as

$$E_T(t) = E_0 \cos(\omega_{\text{NIR}} t) \exp(-i^2/\tau) \exp(i[k_{\text{NIR}}(t) + i\kappa_{\text{NIR}}(t)]L),$$

(1)

with $\omega_{\text{NIR}}$ as the center frequency of the NIR pulse and $\tau$ as a parameter that sets the pulse full width at half maximum (FWHM). The last exponential factor in Eq. (1) contains the propagation constant $k_{\text{NIR}}$ and extinction coefficient $\kappa_{\text{NIR}}$ of the active QW layers over an interaction length $L$ modulated by the terahertz-field across the QWs, which account for FM and AM, respectively. The time-modulated $k_{\text{NIR}}(t)$ and $\kappa_{\text{NIR}}(t)$ are obtained adiabatically from the real and imaginary parts of the complex linear optical susceptibility of the actual QW by varying it parametrically with the modulating electric field.\textsuperscript{5} We use numerically calculated data, which have been proven to be valid by comparing with the experimental data at fixed photon energies. The validity of the adiabatic approach up to a few terahertz and other constraints in applying Eq. (1) to the employed InP-based single QWs have been addressed in Ref. 5.

The terahertz-sideband generation properties of the transmitted light can be analyzed numerically using Eq. (1), which, however, can be developed further to elucidate the effects of AM and FM. The last exponent in Eq. (1) can be separated into AM and FM contributions $\exp[-L\kappa_{\text{NIR}}(t)]$ and $\exp[iLk_{\text{NIR}}(t)]$, respectively. Because we assume a

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single-frequency modulating field with a period \( T = 2\pi/\Omega \), the AM factor, which is also periodic in \( T \), can be expanded in a Fourier series,

\[
\exp[-L\kappa_{\text{NIR}}(t)] = \sum_{n=-\infty}^{\infty} \bar{\kappa}_n e^{in\Omega t},
\]

(2)

where \( \bar{\kappa}_n = (1/T) \int_0^T dt e^{-\kappa(t)} e^{-i\Omega t} \). For the FM, periodic in \( T \) as well, we first expand the imaginary argument of the exponent in a Fourier series,

\[
L\kappa_{\text{NIR}}(t) = \sum_{m=-\infty}^{\infty} \bar{k}_m e^{im\Omega t} = \bar{\kappa}_0 + \sum_{m'=-1}^{\infty} 2|\bar{k}_m'| \cos(m' \Omega t + \phi_{m'}),
\]

(3)

where \( \bar{k}_0 = (1/T) \int_0^T dt Lk_{\text{NIR}}(t) e^{-i\Omega t} \). In Eq. (3), the first term \( \bar{\kappa}_0 \) is the average of the phase shift in \( T \), while the cosine terms induce the FM. At this stage, the FM exponent is expressed as

\[
\exp[iLk_{\text{NIR}}(t)] = e^{i\tilde{k}_0 t} e^{i(\tilde{k}_m t) \cos(m' \Omega t + \phi_{m'})} \cdots \cdots.
\]

(4)

Each factor that contains the cosine in the exponent, which corresponds to index \( m' \) in Eq. (3) can be further expanded into a series as

\[
e^{i(\tilde{k}_m t) \cos(m' \Omega t + \phi_{m'})} = 1 + i(2|\tilde{k}_m'| \cos(m' \Omega t + \phi_{m'})) + \frac{1}{2!}(i(2|\tilde{k}_m'| \cos(m' \Omega t + \phi_{m'})))^2 + \cdots.
\]

We truncate the above series following the explicitly shown terms by assuming \( 2|\tilde{k}_m'| \ll 1 \) and substitute the result back to Eq. (4) to obtain

\[
\exp[iLk_{\text{NIR}}(t)] = e^{i\tilde{k}_0 t} [1 + i(\tilde{k}_1 e^{i\Omega t} + \tilde{k}_2 e^{-i\Omega t})] \cdots \cdots.
\]

(5)

Above, we employed the NBFM theory because it was found that the maximum change in the exponent \( \max[L\Delta k_{\text{NIR}}(t)] \) in this case study is always much smaller than unity, which, in turn, yields \( 2|\tilde{k}_m'| \ll 1 \); the interaction length is \(< 10 \ \mu m \) and the available modulation of the propagation constant is \(< 0.02 \ \mu m^{-1} \) for the maximum sideband conversion over a range of optical wavelengths. It should be noted, however, that if \( \tilde{k}_m' \) exceeds \( \sim 0.2 \) where the interaction length can be made quite large due to a very low extinction coefficient or the modulation of propagation constant can be made very large due to an abrupt change in frequency of the optical properties, the wide-band FM theory should be employed.

The generated sidebands of any order can now be evaluated analytically using Eqs. (2) and (5). By restricting our interest only up to the second order, which is evaluated by accounting for terms up to \( \pm 2\Omega \) in Eqs. (2) and (5) (higher-order terahertz sidebands in practice are extremely weak), we obtain the modulation factor

\[
\exp(i[k_{\text{NIR}}(t) + i\kappa_{\text{NIR}}(t)]L) = e^{i\tilde{k}_0 t} \tilde{k}_2 e^{i\Omega t} + i\tilde{k}_1 e^{i\Omega t} + i\tilde{k}_1 e^{-i\Omega t} + i(\tilde{k}_2 e^{i\Omega t} + \tilde{k}_2 e^{-i\Omega t}) + i(\tilde{k}_1 e^{i\Omega t} + \tilde{k}_1 e^{-i\Omega t}) + i(\tilde{k}_2 e^{i\Omega t} + \tilde{k}_2 e^{-i\Omega t}) + \cdots.
\]

(6)

The exponent \( e^{i\tilde{k}_0 t} \) represents the dc phase shift by the average of the FM. The first line, which accounts for the attenuation of the fundamental carrier frequency, contains only the dc term of the AM. The second and third (fourth and fifth) lines generate \( n = \pm 1 \) \((\pm 2)\) sidebands, where the dominant terms are those in the second (fourth) line. The third (fifth) line may be ignored due to the product of two small coefficients \( \tilde{k}_{0,n} \) and \( \tilde{k}_{0,m} \) \((n \neq m \) (and \( m \) are not zero). The former two terms on the second (fourth) line of Eq. (6) are ascribed only to the AM and the latter two terms on the line are by the FM multiplied by the factor of the fundamental carrier attenuation \( \tilde{k}_0 \). In a sense, the latter two terms are just the result of FM of the already amplitude-modulated fundamental carrier signal. This may be generalized to the higher-order sidebands. In this study, we take only the dominant \( n = \pm 1 \) sidebands [the second line in Eq. (6)] and analyze the effects of AM by the former two and FM by the latter two. The theory introduced in this section may be applied to any terahertz-sideband generation devices as long as the above mentioned constraints meet.

We analyze terahertz-sideband generation of a weak NIR (wavelength \( \sim 1550 \) nm) Gaussian pulse with FWHM of \( \sim 10 \) ps incident normal to a \( \sim 1 \) \( \mu m \) thick InGaAsP-based rectangular multiple QW. A dc bias field \( (F_{dc}) \) and an ac electric field \( (F_{ac}) \), whose frequency may be up to a few terahertz, are applied across the QW's. The stationary extinction coefficient and the propagation constant of the QW layers at the wavelength of \( \sim 1550 \) nm is plotted as a function of \( F_{dc} \) in Fig. 1, which were calculated using the quasi-equilibrium semiconductor Bloch function in the low-density limit that yields complex linear optical susceptibilities. The extinction coefficient is smallest at \( F_{dc} = 0 \), which is the flatband condition, because the NIR wavelength is much smaller than that of the optical transition band edge of the QWs. As \( F_{dc} \) increases the QCSE shifts the optical transition edge to longer wavelength and consequently the extinction coefficient increases drastically up to some \( F_{dc} \) and keeps its value.

FIG. 1. (Color online) Extinction coefficient (solid) and propagation constant (dashed) of multiple layers of InGaAsP single QW at 1550 nm at 300 K as functions of dc bias field.
after the shifted exciton peak energy meets the illuminating NIR wavelength at $F_{dc} \approx 110 \text{ kV/cm}$. In Fig. 1, due to the strong field-dependent line broadening of InP-based QWs, the exciton peak is not well observed. The propagation constant, on the other hand, begins to decrease drastically right before the exciton peak.

In Figs. 2 and 3(a) the powers of the transmitted fundamental carriers and $n=1$ sidebands normalized to the input power at three different modulating field strengths ($F_{ac}$), which are calculated numerically using Eq. (1) are shown. Use of powers instead of amplitudes of the signals is more straightforward in discussing the sideband generation properties. The attenuation of the fundamental carriers in general follows the $F_{ac}$-dependent extinction coefficient curve in Fig. 1, which we found almost equivalent to the $F_{dc}$-dependent $\kappa_0$ on the first line in Eq. (6). The $n=1$ sideband conversion efficiency $\eta$ is shown to increase as $F_{ac}$ increases exhibiting maximum efficiency at an optimum $F_{dc}$.

The dependence of the $n=1$ sideband conversion efficiency $\eta$ on $F_{dc}$ in Fig. 3(a) can be analyzed explicitly using Eq. (6) in terms of AM and FM. Figure 3(b) shows the contributions to $\eta$ at $F_{ac}=10 \text{ kV/cm}$ by the AM (dashed) and the FM (solid) obtained using the respective former and latter two terms on the second line in Eq. (6). One can observe that the addition of the two curves in Fig. 3(b) is identical to the solid in Fig. 3(a) that was obtained using Eq. (1). Figure 3(b) clearly shows that the contributions to the terahertz sidebands by the AM and FM follow the absolute values of the first derivatives (or slopes) of the extinction coefficient and propagation constant, respectively, in Fig. 1. The optimum values of $F_{dc}$ for the maximum contributions due to AM and FM to the terahertz sidebands are $\sim 100$ and $\sim 110 \text{ kV/cm}$, respectively, where the maximum slopes of the two curves are found in Fig. 1. The maximum $\eta$ at $F_{ac}=10 \text{ kV/cm}$ in Fig. 3(a), which is larger than that of the two curves in Fig. 3(b), is, however, found at $F_{dc} \approx 105 \text{ kV/cm}$. This shows that the maximum $\eta$ is not dominated by only either AM or FM effects individually. For instance, the maximum value of $\eta \approx 0.34 \times 10^{-3}$ at $F_{ac}=10 \text{ kV/cm}$ at $F_{dc}=106 \text{ kV/cm}$ in Fig. 3(a) is attributed in part to the contributions of $\eta \approx 0.165 \times 10^{-3}$ due to AM and of $\approx 0.175 \times 10^{-3}$ due to FM. On the other hand, at biases substantially below and above this optimum values, the terahertz sidebands are dominated by AM and FM, respectively, making it obvious that the nearly constant values at high biases are ascribed to FM. It should be remembered that the FM curve in Fig. 3(b) is not the result of purely FM; it is multiplied by the fundamental attenuation coefficient $\kappa_0$, which is also a function of $F_{dc}$.

In conclusion, we presented an analysis of terahertz-sideband generation from the viewpoint of AM and FM. At the optimum bias field, the frequency and AMs contribute almost equally to the maximum first-order sideband conversion efficiency $\eta$. We also found that $\eta$ is dominated by AM and FM, respectively, at biases substantially below and above the optimum values. When the FM effect is very large, the equations should be developed by incorporating the wide-band FM theory in Eq. (4). The employed theoretical approach will facilitate the engineering and optimization of modulator-type sideband-generation devices and wavelength-conversion devices that operate up to a few terahertz, let alone at low frequencies, where the adiabatic approach holds.

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